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Exotic scalar states in the AdS/CFT correspondence

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Abstract: We investigate a family of solutions of type-IIB supergravity which asymptotically approach $AdS_5 \times S^5$ but contain a non-constant dilaton and volume scalar for the five-sphere. These solutions preserve an $\mathrm{SO}(1,3) \times \mathrm{SO}(6)$ symmetry. We discuss the solution in the context of the AdS/CFT correspondence, and we find that as well as running coupling from the nontrivial dilaton, the corresponding field theory has no supersymmetry and displays confinement at least for a certain range of parameters.

Keywords: Confinement, D-branes, Conformal Field Models in String Theory, Supergravity Models.

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Contents

1. Introduction

One of the most interesting aspects of the AdS/CFT correspondence [1, 2, 3] — for a comprehensive review, see ref. [4] — is its potential as a theoretical tool to study real world QCD nonperturbatively [5, 6]. In the context of string theory where the AdS/CFT duality is best understood, simple backgrounds are typically both supersymmetric and conformally invariant. In order to make progress in investigating real world QCD, it is necessary to construct models in which both of these symmetries are eliminated. The initial suggestion [5] in this direction was to compactify a higher dimensional theory with nonsupersymmetric boundary conditions. In this way, ordinary four-dimensional Yang-Mills theory can be studied through an M5-brane construction, and similarly D3-branes can be used to investigate three-dimensional Yang-Mills [5]. This proposal was extensively studied [7, 10, 11, 12, 8, 9] but there are problems in decoupling the additional fermions and scalars in a regime where the supergravity calculations can be trusted. Another promising suggestion [13] was to study D3-branes in nonsupersymmetric string theories. Various investigations [13, 14, 15, 8] have shown that these theories often reproduce log-scaling of the gauge coupling in the ultra-violet. Drawbacks in this case are that connecting the

UV and IR solutions requires considerable fine tuning, and even when this can be done the IR solutions are usually dominated by α' corrections.

Another complementary approach has been to consider new nonsupersymmetric backgrounds within the context of the usual type-IIB superstring theory. One can preserve the Poincaré invariance of the field theory by exciting scalar fields in the supergravity background. This corresponds to exciting scalar operators in the dual Conformal Field Theory (CFT). Such constructions have been presented in refs. [16, 17, 18, 19] — similar supersymmetric constructions may be found in ref. [20] — and it has been found that the dual field theories do indeed exhibit QCD-like behavior. In these examples [16, 17, 18, 19], evidence was found for both confinement and running of the gauge coupling. In this paper, we continue this line of investigation with the examination of a new two-parameter family of solutions of the type-IIB supergravity equations which are asymptotically $AdS_5 \times S^5$ but contain two non-constant scalars: the dilaton and the volume scalar for the five-sphere. In these solutions, supersymmetry and conformal invariance are both broken by the presence of the nontrivial scalars. They also display interesting QCD-like behavior, a running gauge coupling and confinement in the infra-red. The latter is signalled by an area law for Wilson loops and a mass gap in the glueball spectrum, at least for a certain range of the parameters. One property that distinguishes the present solutions from those considered in refs. [16, 17, 18, 19] is that we can demonstrate that they are realized as the throat-limits of asymptotically flat D3-brane spacetimes.

A summary of our key results is as follows. For our new supergravity solutions, we demonstrate that: (i) The dual field theories exhibit both confinement in the infrared as well as running of the gauge coupling, for a certain range of the parameters. (ii) Unfortunately, only a limited sector of the low-energy physics is insensitive to the region of strong curvatures at the center of the supergravity solutions. (iii) Further, there is no obvious way to decouple the fermions and scalars in the dual field theory, essentially because the supergravity solutions contain a single dimensionful parameter. (iv) The long-range supergravity fields have a standard interpretation in terms of the expectation values of operators in the dual field theory. In particular, one finds $\langle T_{ab}\rangle = 0$. (v) In conjunction with (iv), the nontrivial scalar field profiles are *not* triggered by the irrelevant operator $\mathcal{O}_8$, as suggested in refs. [16, 17]. All of these results should be generic for the Poincaré invariant class of solutions considered in this approach to constructing QCD-like field theories [16, 17, 18, 19].

The remainder of the paper is organized as follows: In section 2, we describe the new two-parameter family of solutions of the ten-dimensional supergravity equations. In section 3, we show that this geometry produces an area law for Wilson Loops. Section 4 addresses the issue of a mass gap, and in section 5, we discuss our results and consider the dual field theory interpretation of these solutions.

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2. Background solutions

In this paper we will investigate a two-parameter family of ten-dimensional solutions of type-IIB supergravity. The Einstein-frame metric is given by:

$$ds^2 = H^{-1/2}\left(1+\frac{2\omega^4}{r^4}\right)^{\delta/4}(-dt^2+dx^2+dy^2+dz^2)+ \\ +H^{1/2}\left(1+\frac{2\omega^4}{r^4}\right)^{\frac{2-\delta}{4}}\left[\frac{dr^2}{(1+\omega^4/r^4)^{5/2}}+\frac{r^2\, d\Omega_5^2}{(1+\omega^4/r^4)^{1/2}}\right], \qquad (2.1)$$

where

$$H=\left(1+\frac{2\omega^4}{r^4}\right)^{\delta}-1\,.$$

These solutions are parameterized by two constants: ω which has the dimensions of length, and δ which is dimensionless. We will assume without loss of generality that ω is positive, and requiring the metric to be real requires that δ is also positive. The dilaton is given by:

$$e^{2\phi}=e^{2\phi_0}\left(1+\frac{2\omega^4}{r^4}\right)^{\Delta}, \qquad (2.2)$$

where ϕ_0 is an arbitrary constant, and the exponent Δ must satisfy

$$\Delta^2+\delta^2=10\,. \qquad (2.3)$$

The "electric" components of the RR five-form are determined by the potential

$$A^{(4)}_{\rm elec}=-\frac{1}{4\kappa}H^{-1}\; dt\, dx\, dy\, dz\,, \qquad (2.4)$$

where $\kappa^2=8\pi G$ with G being the ten-dimensional Newton's constant. The remaining "magnetic" components are determined by requiring $F^{(5)}=*F^{(5)}$. One can express the result as

$$F^{(5)}=\frac{2\delta\omega^4}{\kappa}\left(\varepsilon(S^5)+*\varepsilon(S^5)\right), \qquad (2.5)$$

where $\varepsilon(S^5)$ is the volume form on a round five-sphere of unit radius.

In the following, it will also be useful to have the string-frame metric, which is given by: $G_{MN}=e^{(\phi-\phi_0)/2}g_{MN}$ where G_{MN} and g_{MN} are the string-frame and Einstein-frame metrics, respectively. Hence the metric in the string-frame becomes

$$dS^2 = H^{-1/2}\left(1+\frac{2\omega^4}{r^4}\right)^{\frac{\Delta+\delta}{4}}(-dt^2+dx^2+dy^2+dz^2)+ \\ +H^{1/2}\left(1+\frac{2\omega^4}{r^4}\right)^{\frac{\Delta-\delta+2}{4}}\left(\frac{dr^2}{(1+\omega^4/r^4)^{5/2}}+\frac{r^2\, d\Omega_5^2}{(1+\omega^4/r^4)^{1/2}}\right). \qquad (2.6)$$

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Asymptotically at large r, the geometry approaches $AdS_5 \times S^5$ where the radius of curvature of both the AdS factor and the sphere is given by

$$L^4 = 2\delta\omega^4 \,. \tag{2.7}$$

The usual $AdS_5 \times S^5$ solution can be reproduced as a (singular) limit: set $\delta = 1$ and then take $\omega \to 0$, while rescaling the boundary coordinates (t, x, y, z) and the inverse string tension α'.

In either the Einstein-frame (2.1) or string-frame (2.6), the surface $r = 0$ is time-like, i.e. the tangent space contains a time-like direction. We also note that proper distance is finite in going from $r = 0$ to finite r along a radial line. Investigating curvature invariants such as R or $R_{KLMN}R^{KLMN}$, one finds that they diverge as $r \to 0$, and hence $r = 0$ is a naked singularity. Again, this result applies for both the Einstein- and string-frame metrics. In the latter case, the divergence of the curvature invariants signals the breakdown of the α' expansion, and so one can not trust the precise form of the solution in this vicinity as a string theory background. One can also see the singularity at $r = 0$ by checking simpler invariants involving the dilaton field. For example, as $r \to 0$,

$$\nabla_M\phi\,\nabla^M\phi \;\longrightarrow\; \frac{\Delta^2}{2\omega^2}\left(\frac{2\omega^4}{r^4}\right)^{\frac{10-\delta-\Delta}{4}}\,, \tag{2.8}$$

using the string-frame metric (2.6). Setting $\Delta = 0$ in the exponent above yields the result for the Einstein-frame (2.1). This expression diverges for any of the allowed values of δ, independent of the sign of Δ. This divergence also indicates the breakdown of the α' expansion near $r = 0$. One finds that the Ricci scalar displays precisely the same divergent behavior as $r \to 0$. Of course, the result in eq. (2.8) vanishes for $\Delta = 0$ (i.e. $\delta = \sqrt{10}$) since the dilaton (2.2) is constant.

Turning to eq. (2.2), one sees that for positive Δ the string coupling e^ϕ diverges as $r \to 0$. Hence in this case, the string loop expansion is uncontrolled near $r = 0$, providing an additional reason that the detailed form of solution can not be trusted in this vicinity. On the other hand, for negative Δ, the string coupling vanishes as $r \to 0$.

The new solutions break all of the supersymmetries as can be seen by the transformation of the supergravity spinors [21]

$$\delta\lambda = \frac{1}{2}\gamma^M\partial_M\phi\,\epsilon^* = \frac{1}{2}\gamma^r\partial_r\phi\,\epsilon^* \,. \tag{2.9}$$

Hence there can be no Killing spinor solutions for $\delta\lambda = 0$. Of course, this variation vanishes in the case of a constant dilaton (i.e. $\Delta = 0$), and so one would have to consider the gravitino variation. We will not pursue this case further here.

Given the pathological behavior at $r = 0$, a complete understanding of these solutions and the dual field theory requires a full knowledge of string theory. In many cases, however, we will be able to extract the interesting physics while only considering regions where supergravity is reliable. We can trust the supergravity approximation as long as curvature scales are large compared to the string scale, and the string coupling is small. As a prerequisite then, we will require in the asymptotic region

$$\frac{L^4}{\alpha'^2} = \frac{2\delta\omega^4}{\alpha'^2} \gg 1 \qquad \text{and} \qquad e^{\phi_0} \ll 1\,. \tag{2.10}$$

Recall that the former quantity is related to the 't Hooft coupling

$$\lambda = g_{YM}\sqrt{N} = \frac{L^2}{\alpha'}\,. \tag{2.11}$$

in the ultraviolet regime for the U(N) gauge theory.

Since the present family of solutions differ from $AdS_5 \times S^5$ by virtue of nontrivial excitations of the dilaton, the volume scalar for the five-sphere and components of the five-dimensional metric, one expects that in the dual $\mathcal{N} = 4$ super Yang-Mills theory, expectation values for various operators, such as $\mathrm{Tr}(F^2)$, have been turned on. From the field theory point of view, these expectation values break the conformal invariance and the supersymmetry. This in turn produces a running of the gauge coupling and confinement in the infra-red, as we will demonstrate below.

3. Quark-antiquark potential

The first test of confinement that we carry out is the calculation of the quark-antiquark potential [22, 23, 24, 12]. The standard procedure is to minimize the Nambu-Goto action for a fundamental test string in the background given above. The action is,

$$S = \frac{1}{2\pi\alpha'}\int d^2\sigma \sqrt{-\det G_{MN}\,\partial_a x^M \partial_b x^N} \tag{3.1}$$

where G_{MN} denotes the string frame metric (2.6).

We are interested in finding a static solution to the equations of motion so it is convenient to set $\sigma^0 = t$ and $\sigma^1 = x$, and assume $y = z = 0$. With these choices, the action becomes

$$S = \frac{1}{2\pi\alpha'}\int dt\, dx\, n(r)\left[1 + m(r)\left(\frac{dr}{dx}\right)^2\right]^{1/2}, \tag{3.2}$$

with

$$\begin{aligned} n(r) &= H^{-1/2}\left(1 + \frac{2\omega^4}{r^4}\right)^{\frac{\delta+\Delta}{4}} \\ m(r) &= H\frac{(1 + 2\omega^4/r^4)^{\frac{1-\delta}{2}}}{(1 + \omega^4/r^4)^{\frac{5}{2}}}\,. \end{aligned} \tag{3.3}$$

If $n(r)$ possesses a global minimum at some position r_{min} then a long string situated at this minimum is a solution of the equations of motion. It is straightforward to show that $n(r)$ does have a global minimum located at

$$r^4_{min} = \frac{2\omega^4}{\left(\frac{|\Delta|+\delta}{|\Delta|-\delta}\right)^{1/\delta} - 1} \tag{3.4}$$

for $\Delta > 0$ and $\delta \leq \sqrt{5}$. (Above, we have added the absolute values in this formula for later considerations.) The idea here is that when the end-points of the string are widely separated at the boundary of the AdS space, the string will fall into the interior of the space and settle at r_{min} [17]. So evaluating the action for large separations, one would find a linear quark anti-quark potential,

$$V(\Delta x) \simeq \frac{n(r_{min})}{2\pi\alpha'}\,\Delta x = \frac{\Sigma^{\frac{\delta-|\Delta|}{4\delta}}}{2\pi\alpha'\sqrt{1-\Sigma}}\,\Delta x\,, \tag{3.5}$$

with

$$\Sigma = \frac{|\Delta| - \delta}{|\Delta| + \delta}\,,$$

where the coordinate distance Δx between the string endpoints corresponds to the separation between the quark and anti-quark in the background geometry of the dual field theory. Further using the above definition of the gauge coupling (2.11), the QCD string tension can be written as

$$T_{QCD} \simeq \frac{n(r_{min})\lambda}{2\pi L^4}\,. \tag{3.6}$$

The linear quark-antiquark potential is then evidence for the confinement of electric charge in the dual gauge theory.

We should note that the validity of our calculation depends on r_{min} being far from the origin since, as discussed above, the supergravity approximation is not valid in this vicinity. From eq. (3.4), we find that r_{min} is of the order of ω for small values of δ, and that $r_{min} \to 0$ as $\delta \to \sqrt{5}$. For $\delta > \sqrt{5}$ or $\Delta < 0$, $n(r)$ has no minima at finite r. Rather it monotonically decreases to zero at $r = 0$ for this parameter regime.

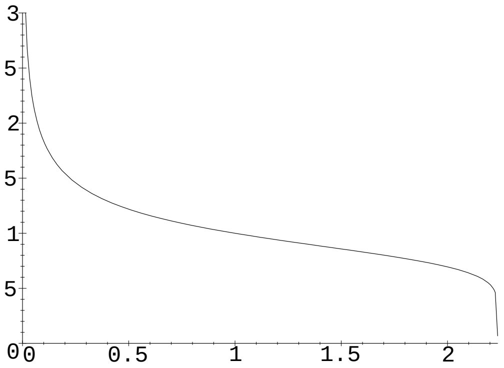

Figure 1: Plot of r_{min}/L (vertical axis) versus δ. Here L is the characteristic curvature scale of the asymptotic AdS region, given by eq. (2.7). Note that as δ approaches zero, r_{min} is getting large and as $\delta \to \sqrt{5}$, $r_{min} \to 0$.

Plotting the ratio of r_{min} and the asymptotic AdS radius L in fig. 1, we see that for $\delta < 2$ (and $\Delta > 0$), r_{min} is roughly of the order of L, which is assumed to be large. Hence we expect that our results are reliable for this parameter range. For $\delta > 2$ or $\Delta < 0$, we can't make any quantitative conclusions about the quark-antiquark potential without going beyond the supergravity approximation. One might observe the trend, however, that the QCD string tension (3.6) is decreasing as δ approaches $\sqrt{5}$ (and hence, at the same time, approaches Δ). Note that curiously the limiting value is $T_{QCD}|_{\delta=\sqrt{5}} = 1/(2\pi\alpha')$, which is precisely the fundamental string tension in flat empty space.

We checked the previous expectations with figs. 2 and 3. Fig. 2 shows that $(\nabla\phi)^2$ is well behaved at r_{min} for $\delta < 2$, but that for larger values, the α' expansion is breaking down in the vicinity of r_{min}. Hence our results calculated in the supergravity approximation are not reliable for these large values of δ. Similarly, fig. 3 shows that over the range $0 < \delta < 2$ (with Δ positive) the string coupling at r_{min} is roughly the order of the asymptotic value, which is taken to be small. However, for larger values of δ, the position of the test string strays into regions where the string loop expansion is uncontrolled, and hence the details of the supergravity solution are unreliable.

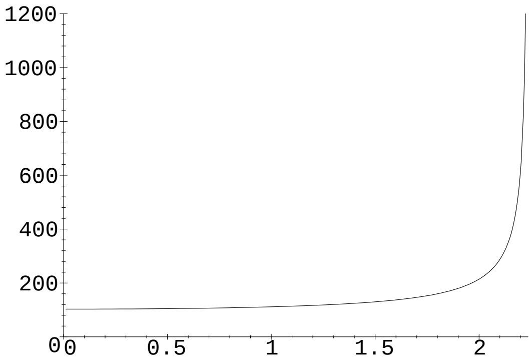

Figure 2: Plot of $L^2(\nabla\phi(r_{min}))^2$ (vertical axis) versus δ. Here $\phi(r)$ is the background dilaton (2.2), and L is the asymptotic AdS scale (2.7). The figure suggests that the α' expansion is breaking down at r_{min} for $\delta > 2$.

We might attempt to determine the monopole-antimonopole potential [25] by repeating the above calculation with the action of a D-string which is given by

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma \, e^{-\phi} \sqrt{-\det G_{MN}\, \partial_a x^M \partial_b x^N}\,. \tag{3.7}$$

Assuming the static gauge as above, the action has precisely the same form as in eq. (3.2) except that the function $n(r)$ is replaced by

$$N(r) = H^{-1/2} \left(1 + \frac{2\omega^4}{r^4}\right)^{\frac{\delta-\Delta}{4}} \,. \tag{3.8}$$

Now for $\Delta > 0$ as in the case of interest above, $N(r)$ has no extrema for any real values of r. Rather the function $N(r)$ is a monotonically decreasing function of r as the string approaches the singularity. If one were to use this result, it would appear that the most energetically favorable situation is for the string to fall all the way to the singularity where $N(r)$ vanishes and then to run along the singularity for a distance Δx before returning to the asymptotic region of the space-time. This is possible because at $r = 0$ it costs no energy to separate the monopoles and the interpretation is that the magnetic charge is screened [15]. However as discussed, we can not trust this conclusion since the supergravity approximation breaks down in the vicinity of $r = 0$.

However, one might also consider the monopole-antimonopole potential [15, 11, 12] for $\Delta < 0$. In this case, $N(r)$ does have a global minimum whose position is given by eq. (3.4) for $\delta \leq \sqrt{5}$, and so a linear potential of the form (3.5) arises for the monopoles in this parameter range. Further note that in this case, $n(r)$ decreases monotonically to zero at $r = 0$, which is suggestive of screening of electric charge. Hence the quarks and monopoles have interchanged roles for $\Delta < 0$. Mathematically one can see that this result arises because changing the sign of Δ (i.e. $\Delta \to -\Delta$) precisely interchanges the form of $n(r) \leftrightarrow N(r)$ and so trades the fundamental test string action for that of a D-string. This is to be expected since as can be seen from eq. (2.2), $\Delta \to -\Delta$ implements the $\mathrm{SL}(2,\mathbb{Z})$ transformation $e^{\phi} \to e^{-\phi}$. Hence one might have expected that the physics of the quarks and monopoles should be interchanged in the two complementary parameter ranges of positive and negative Δ.

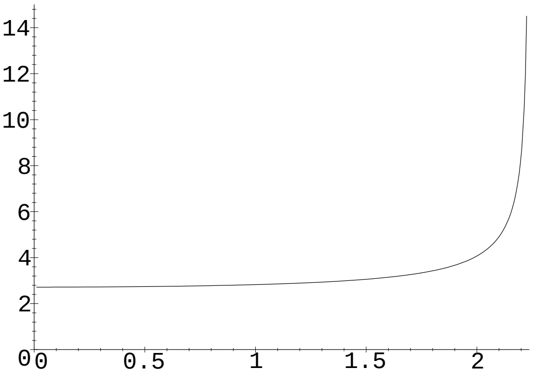

Figure 3: Plot of the $e^{\phi(r_{min})-\phi_0}$ (vertical axis) versus δ. The vertical axis gives ratio of the string coupling evaluated at r_{min} to that in the asymptotic AdS region. The plot is made for positive $\Delta = \sqrt{10-\delta^2}$ in eq. (2.2). In this case, the figure suggests that the string loop expansion will become uncontrolled for $\delta > 2$.

4. Mass gap and glueballs

As argued by Witten [5], a second useful criterion in establishing confinement in the dual gauge theory is the presence of a mass gap. To demonstrate the existence of a mass gap in the present case we begin by calculating the mass spectrum for scalar glueballs. The prescription for performing such a calculation was originally given in ref. [7]. The strategy is to examine the linearized equation of motion for a scalar field fluctuation η propagating in the supergravity background, and determine the spectrum of $m^2 = -k^2$ where k is the four-momentum of the scalar field in the boundary directions spanned by t, x, y and z. Making the ansatz $\eta = h(r)e^{ikx}$, the equation of motion takes the form of a linear second-order ODE for the radial profile $h(r)$. In this section, we will consider two scalars, the axion (i.e. the RR scalar) and the dilaton. The linearized equation of motion for the axion may be written as

$$e^{-(\phi-\phi_0)/2}\left(\nabla^2_{\mathrm{Einstein}}C + 2(\partial_M\phi)(\partial^M C)\right) = \nabla^2_{string}C = 0\,, \tag{4.1}$$

while that for the dilaton is

$$\nabla^2_{\mathrm{Einstein}}\varphi = 0\,. \tag{4.2}$$

Actually the latter deserves some explanation. Because of the nontrivial dilaton profile in the background solution, the linearization of the dilaton equation of motion (i.e. $\nabla^2_{\rm Einstein}\phi = 0$) includes terms involving both the dilaton fluctuation φ and the metric fluctuation h_{MN}. In the Einstein frame, the complete equation is

$$\nabla^2\varphi + \nabla^N\left(\left(h_{MN} - \frac{1}{2}g_{MN}h^P_P\right)\nabla_M\phi\right) = 0\,. \tag{4.3}$$

However the second term in eq. (4.3) can be eliminated with the gauge choice

$$h_{Mr} - \frac{1}{2}g_{Mr}h^P_P = 0\,, \tag{4.4}$$

in which case the equation reduces to eq. (4.2). Note that this does not mean that there is no perturbation of the metric which accompanies the dilaton fluctuation. Certainly there is no gauge freedom which can eliminate φ terms from the linearized Einstein's equations. However the preceding argument does show that for the judicious choice of gauge (4.4) above, analyzing the simple scalar equation (4.2) is sufficient to determine the spectrum of glueballs and there is no need to solve the linearized Einstein equations.

Thus from eq. (4.1),the axion propagates as a free field on the string-frame geometry, while from eq. (4.2), the dilaton fluctuation does so on the Einstein-frame geometry. We can use this to our advantage in calculating the spectra of the corresponding glueballs simultaneously. Notice that setting the parameter $\Delta = 0$ reduces the string-frame metric (2.6) to the Einstein-frame metric (2.1). Thus we can analyze the axion equation (4.1) and by setting $\Delta = 0$ obtain the results for the dilaton for free![1] Making the ansatz $C = h(r)\,e^{ikx}$, the axion equation (4.1) becomes

$$\begin{aligned}
\Box_{\rm String}C &= \frac{\partial^2 h(r)}{\partial r^2} + f(r)\frac{\partial h(r)}{\partial r} + m^2 g(r)h(r) = 0\,,\\
f(r) &= \frac{5r^4 + 2\omega^4\left(1 - 4\Delta\right)}{r\left(r^4 + 2\omega^4\right)}\,,\\
g(r) &= \frac{r^{10}\left(\left(1 + 2\omega^4/r^4\right)^\delta - 1\right)\left(1 + 2\omega^4/r^4\right)^{(1-\delta)/2}}{\left(r^4 + \omega^4\right)^{5/2}}\,.
\end{aligned} \tag{4.5}$$

One can check that the dilaton equation (4.2) agrees with (4.5) upon setting $\Delta = 0$.

Applying the methods of refs. [8, 9], we change variables to $r = \omega e^y$ and redefine the field $h(y) = \alpha(y)\psi(y)$ so that the equation is Schrödinger-like:

$$0 = -\psi(y)'' + V(y)\,\psi(y)\,, \tag{4.6}$$

where the effective potential is given by

$$V(y) = -\frac{\alpha(y)''}{\alpha(y)} - f(y)\frac{\alpha(y)'}{\alpha(y)} - m^2 g(y)\,, \tag{4.7}$$

[1]It should be emphasized that this is a purely formal statement. In our solution, choosing $\Delta = 0$ uniquely fixes the parameter δ by eq. (2.3), while in the dilaton equation (4.2), δ is to be understood as a free parameter.

with

$$\alpha(y) = \left(e^{4y} + 2\right)^{-\Delta/4-1/2} e^{\Delta y}$$

The primes denote differentiation with respect to y.

The potential $V(y)$ has the following asymptotic behavior:

$$\begin{aligned} V(y \gg 0) &\sim 4 - a\,\omega^2 m^2 e^{-2y}\,, \\ V(y \ll 0) &\sim 3\Delta^2 - b\,\omega^2 m^2 e^{2(5-\delta)y}\,, \end{aligned} \tag{4.8}$$

where a and b are positive constants.

Notice that naively for $y \ll 0$ the m^2 term in the potential would diverge to $-\infty$ for $\delta > 5$. However, from eq. (2.3), one has the restriction that $\delta \leq \sqrt{10}$ and hence the potential asymptotically approaches a positive constant for both $y \to \pm\infty$. Hence we can expect that eq. (4.6) will yield normalizable zero-energy bound states when the potential is appropriately adjusted by tuning m^2, which appears as a parameter in the former. We can obtain the glueball spectrum in the WKB approximation following ref. [8]. Our potential has classical turning points located at

$$\begin{aligned} y_+ &= \frac{1}{2}\log\left(\frac{\omega^2 m^2 \delta}{2}\right)\,, \\ y_- &= -\frac{1}{2(5-\delta)}\log\left(\frac{2^{\frac{1}{2}(\delta+1)}\omega^2 m^2}{\Delta^2}\right)\,. \end{aligned} \tag{4.9}$$

Now using the prescription and notation of [9], we find that within the WKB approximation

$$\begin{aligned} m^2 &= \frac{\pi^2}{\omega^2 \xi(\delta)^2} n \left(n + 1 + \frac{\Delta}{5-\delta}\right) + O(1) \\ \xi(\delta) &= \int_{-\infty}^{\infty} dy\,\sqrt{g(y)} \qquad \text{and} \qquad n \geq 1\,, \end{aligned} \tag{4.10}$$

where $g(y)$ is given in eq. (4.5). Note that $\xi(\delta)$ is a number of order one — more precisely it rises monotonically from zero to approximately 2.5 as δ ranges from 0 to $\sqrt{10}$.

This result (4.10) is reliable as long as the bound state wave function $\psi(y)$ has no or little support in the region near the singularity, i.e. $y \to -\infty$, where the supergravity approximation breaks down. In eq. (4.9), the inner turning point will become large and negative when ωm becomes large. From eq. (4.10) and the following discussion, one sees that the latter will occur for small values of δ, and so in this case, we cannot reliably conclude that there is a mass gap. Note that for a generic value of δ, ωm will also become large when $n \gg \xi(\delta)$. In this case, we can conclude that there is a mass gap, however, we do not have a reliable calculation of the mass spectrum for very large values of n — where ordinarily the WKB approximation is most reliable. There is also a problem as $\delta \to \sqrt{10}$, for which $\varDelta \to 0$ and hence from eq. (4.9), $y_- \to -\infty$. Thus $\delta \sim \sqrt{10}$ would be another range where we could not reliably conclude that there is a mass gap.

However, for most values of δ and either sign of $\varDelta$, we have a reliable calculation of the mass spectrum corresponding to the operator $\mathrm{Tr}(F \wedge F)$, and we may conclude that there is a mass gap in the spectrum of these 0^{-+} glueballs of the dual gauge theory.

Finally we turn to the dilaton. We were to reduce the above analysis for the axion to that for the dilaton by setting $\varDelta = 0$. However, we immediately recognize that there is a problem, in that precisely for this choice the inner turning point is $y_- = -\infty$, i.e. $r = 0$. Hence the potential doesn't confine the wave-function away from the singularity, and so the results in eq. (4.10) are not trustworthy. Hence within the supergravity approximation, we cannot determine whether or not there is a mass gap in this case.

As a final comment, we also note that a similar problem arises in analyzing the spin-two glueballs associated with the linearized fluctuations of the five-dimensional (Einstein) metric. In this case it also found that the effective potential for the metric fluctuations does not confine the wave-function away from the singularity, and hence we are again unable to determine if there is a mass gap for these states or not.

5. Discussion

In this paper we have described a new two parameter family of solutions to type-IIB supergravity which are asymptotic to $AdS_5 \times S^5$, but they contain two non-constant background scalars: the dilaton and the volume scalar for the five-sphere. These nontrivial scalar fields serve to break both the supersymmetry and the conformal invariance of the background. However, an $\mathrm{SO}(1,3) \times \mathrm{SO}(6)$ symmetry is preserved. That is Lorentz invariance in the boundary directions and rotational invariance of the five-sphere. Further the supergravity solutions contain a naked time-like singularity at $r = 0$. Despite this singularity, we can still reliably show within the supergravity approximation, that at least for a certain range of the parameters, the field theories dual to these solutions exhibit both confinement in the infrared as well as running of the gauge coupling. We now discuss a number of aspects of the physics of these supergravity solutions and the dual field theories.

5.1 Gauge theory interpretation

It is interesting that the new solutions only contain a single dimensionful parameter ω, while the second free parameter is the dimensionless exponent δ. Combined these two parameters set the AdS radius as given in eq. (2.7). Similarly the glueball mass spectrum (4.10) is determined by ω multiplied by some more complicated function of δ. Below we will also see that ω and δ combine to determine the expectation values

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of certain operators in the dual gauge theory. Given these expectation values, it is natural to expect that masses of the form $f(\delta)/\omega$ are induced for the field theory fermions and scalars, and the QCD-like properties would then follow. Note that all of the scales, e.g., glueball masses and fermion masses, described above are tied to a single dimensionful parameter ω. This means that there is no natural way to suppress the fermions and scalars in the theory. Thus one would expect that there are fermionic glueballs with masses of the same order as those for the scalar glueballs in eq. (4.10) i.e. one could calculate glueball masses for the linearized gravitino equations as in section 4. This behaviour can be contrasted with Witten's proposed model for pure Yang-Mills [5]. There the compactification radius introduces a second dimensionful scale independent of the AdS radius, and a small compactification radius naturally induces large fermion masses. In any event, our model does not describe ordinary nonsupersymmetric Yang-Mills theory in four dimensions, rather it would be Yang-Mills theory coupled to *adjoint* fermions and scalars with masses. However, it provides an interesting frame work in which to study confinement in four dimensional gauge theories.

We showed that for $0 < \delta < 2$ and $\Delta > 0$, the new solutions give rise to a linear quark-antiquark potential, which may be taken as evidence for confinement of electric charge. By flipping the sign of Δ (which amounts to an $\mathrm{SL}(2,\mathbb{Z})$ transformation, sending $e^{\phi} \to e^{-\phi}$), we obtain a solution for which the monopoles of the dual field theory are confined.

As further evidence for confinement, we also demonstrated that for all values of the parameter δ, except near $\delta \sim 0$ or $\delta \sim \sqrt{10}$, there is a mass gap for fluctuations of the RR scalar. In the dual gauge theory, this corresponds to a mass gap in the spectrum of the 0^{-+} glueballs. These calculations are reliable because in the corresponding wave equation, the effective potential provides a repulsive barrier which shields the singularity. The same effect has been observed previously for certain black hole solutions in string theory [28]. On the other hand, no such barrier appears in the linearized wave equations for the dilaton or metric fluctuations. Hence determining the spectrum of these fields would require understanding how the type-IIB string theory resolves the singularity (if this actually occurs — see below).

In ref. [15], a linear quark-antiquark potential and a discrete glueball spectrum was also found for a range of parameters while investigating type-0 strings in the limit of small radius. In the special case that the bulk tachyon is constant in these solutions the relevant equations of motion should reduce to those considered here and in refs. [16]–[19] and so the small radius behavior would be the same.

The running of the gauge coupling is simply a consequence of the dilaton being non-constant. The Yang-Mills coupling is related to that in the string theory by $g^2_{YM} = 2\pi e^{\phi}$ [1]. Further one has the UV/IR relation [26, 27] relating supergravity degrees of freedom at large (small) radius to those in the field theory at high (low) energy. One can interpret the asymptotic approach of the dilaton to a constant value, i.e. $e^{\phi} - e^{\phi_0} \propto 1/r^4$, as the flow of the gauge theory to a UV-stable fixed point [16]. Further this particular form for the asymptotic behavior of the dilaton, which is dictated by the supergravity equations of motion, can be translated into a universal result for the gauge theory beta function [16]

$$\left.\frac{\partial\beta}{\partial g_{YM}}\right|_{g_{YM}=g^*_{YM}} = -4\,, \tag{5.1}$$

where g^*_{YM} is the value of the gauge coupling at the fixed point. One can confirm that the higher derivatives of the beta function are non-universal [16], depending on the parameter Δ for the present solutions.

Further considerations of universal behavior in asymptotically AdS solutions with nontrivial scalar fields appear in ref. [29]. To make contact with the discussion there, we must first identify the five-dimensional Einstein metric $g^{(5)}_{\mu\nu}$ from eq. (2.1) with

$$ds^2 = e^{-\frac{10}{3}\chi}\, g^{(5)}_{\mu\nu} dx^\mu dx^\nu + L^2 e^{2\chi}\, d\Omega_5^2\,. \tag{5.2}$$

where L is the asymptotic radius given in eq. (2.7). In accord with the prediction of ref. [29], one finds that to leading order near the singularity this metric takes the form

$$g^{(5)}_{\mu\nu} dx^\mu dx^\nu \propto d\rho^2 + \left(\frac{\rho}{\omega}\right)^{1/2} \left(-dt^2 + dx^2 + dy^2 + dz^2\right). \tag{5.3}$$

where ρ is a new radial coordinate. The prediction of this universal behavior relied on being able to ignore any potential terms appearing in the scalar equations of motion near the singularity, which is valid for polynomial potentials. In fact, the volume scalar χ has an exponential potential, as can be seen from its equation of motion [17]

$$\nabla^2\chi = \frac{4}{L^2}\left(e^{-16\chi/3} - e^{-40\chi/3}\right). \tag{5.4}$$

The authors of ref. [29] recognize that an exponential potential could result in violations of the universal behavior considered there. However, this does not seem to be the case for the present supergravity solutions. In fact one can easily show that the potential terms are suppressed as $r \to 0$. Further, as already discussed, we found a linear quark-antiquark potential for a "large" range of the exponents appearing in our solutions, which is in accord with the analysis in ref. [29]. We did not, however, find the universal glueball spectrum with a mass gap arising from their calculations. The obvious reason for this discrepancy is that their universal spectrum arises for a minimally coupled massless scalar propagating on the five-dimensional Einstein metric. In the present case of type-IIB supergravity, there is not obviously any such scalar. Instead we considered the RR scalar (4.1), which propagates on the ten-dimensional string metric, and the dilaton (4.2), which propagates on the ten-dimensional Einstein metric. We only found clear evidence of a mass gap in the former case.

5.2 Asymptotically flat supergravity solutions

The family of supergravity solutions presented here differ from those in refs. [16, 17] since the present geometries can be realized as the throat limits of D3-brane metrics which are asymptotically flat. Consider the following asymptotically flat solutions with the Einstein-frame metric given by

$$
\begin{aligned}
ds^2 &= F^{-1/2}\left(-f^{\delta-\alpha_1-\alpha_2-\alpha_3}dt^2 + f^{\alpha_1}dx^2 + f^{\alpha_2}dy^2 + f^{\alpha_3}dz^2\right) + \\
&\quad + F^{1/2} f^{\frac{2-\delta}{4}}\left[\frac{dr^2}{(1+\omega^4/r^4)^{5/2}} + \frac{r^2\, d\Omega_5^2}{(1+\omega^4/r^4)^{1/2}}\right], \qquad (5.5)
\end{aligned}
$$

where

$$
F = \left(f^{\delta}-1\right)\beta^2 + 1 \qquad \text{and} \qquad f = 1 + \frac{2\omega^4}{r^4}\,.
$$

The dilaton is given by

$$
e^{2\phi} = e^{2\phi_0}\, f^{\Delta}\,, \qquad (5.6)
$$

while the RR five-form has $F^{(5)}_{mag} \propto (\beta^2-1)\,\varepsilon(S^5)$. Here, ϕ_0 is an arbitrary constant, while $\beta \geq 1$ and the exponents satisfy

$$
\Delta^2 + \frac{5}{2}\delta^2 - 4\delta(\alpha_1+\alpha_2+\alpha_3) + 4(\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3) = 10\,. \qquad (5.7)
$$

As before, we will also assume that $\omega > 0$ and $\delta > 0$. One may think of this five-parameter solution as describing a non-extremal D3-brane dressed with extra scalar "hair" associated with exciting the dilaton field and world-volume components of the metric. Note that as a result of this hair, the surface $r = 0$ is generically singular. An exception to this rule is the choice $\Delta = 0$, $\alpha_1 = \alpha_2 = \alpha_3 = 1$ and $\delta = 2$ for which the solution reduces to the standard near-extremal D3-brane. The solution (5.5) was constructed by applying an appropriate set of U-duality transformations [30] to a "hairy" vacuum solution of ref. [31].

The corresponding near-horizon solution is constructed by scaling

$$
r \to \frac{r}{\beta}\,, \qquad \omega \to \frac{\omega}{\beta}\,, \qquad (5.8)
$$

with $\beta \to \infty$. In this decoupling limit, $f \to f$ and so the only change in the form of the solution is that $F \to \beta^2 H$ with $H = f^{\delta} - 1$ as in eq. (2.1). Hence the metric (5.5) becomes $ds^2 = \widetilde{ds}^2/\beta$ where

$$
\begin{aligned}
\widetilde{ds}^2 &= H^{-1/2}\left(-f^{\delta-\alpha_1-\alpha_2-\alpha_3}dt^2 + f^{\alpha_1}dx^2 + f^{\alpha_2}dy^2 + f^{\alpha_3}dz^2\right) + \\
&\quad + H^{1/2} f^{\frac{2-\delta}{4}}\left[\frac{dr^2}{(1+\omega^4/r^4)^{5/2}} + \frac{r^2\, d\Omega_5^2}{(1+\omega^4/r^4)^{1/2}}\right], \qquad (5.9)
\end{aligned}
$$

while the dilaton (5.6) remains unchanged. One may easily verify that in the limit $r \to \infty$, eq. (5.9) approaches the product metric on $AdS_5 \times S^5$ with the radius of curvature given as in eq. (2.7). This solution is an extension of that studied in the present paper because it is no longer Lorentz invariant in the world-volume directions. Lorentz invariance is restored by setting $\alpha_1 = \alpha_2 = \alpha_3 = \delta/4$, in which case one recovers precisely the solution in eq. (2.1). Similarly the asymptotically flat extension of the latter solution is produced by making this choice of exponents in eq. (5.5) — note that eq. (5.7) agrees with eq. (2.3) in this case. The new near-horizon solution may also be regarded as a generalization of those found in ref. [32] by the addition of the dilaton hair.

5.3 Dual interpretation of long-range AdS fields

We now return to the dual field theory interpretation of these supergravity solutions. By examining the precise way in which the solution approaches $AdS_5 \times S^5$ asymptotically, one may determine the expectation values of various chiral operators in the CFT [33]. In considering solutions of the linearized equations of motion around AdS space, one can classify the solutions as modes which are nonsingular on the interior of AdS, and those which are nonsingular at the boundary. For example [3], a massive scalar field satisfying $\nabla^2\,\delta\psi = m^2\,\delta\psi$ has asymptotic modes

$$\delta\psi_\pm \simeq \left(\frac{r}{L}\right)^{\lambda_\pm} \delta\psi_{0\pm}(x^a)\,, \qquad \text{with } \lambda_\pm = -2 \pm \sqrt{4+m^2L^2}\,, \tag{5.10}$$

for asymptotically horospheric coordinates on AdS_5. The modes $\delta\psi_+$ smoothly extend over the interior of the anti-de Sitter space, but produce singularities at the boundary. The functions $\delta\psi_{0+}$ may be associated with source currents for the CFT in calculating correlation functions [3]. The modes $\delta\psi_-$ are nonsingular at the AdS boundary, but extend to solutions which are singular on the interior of the space. The functions $\delta\psi_{0-}$ then yield the corresponding expectation values, as may be seen from test probes moving in the supergravity spacetime [33, 34]. As stressed in ref. [35], the fact that these modes become singular (i.e. reach large values) in the interior of AdS means that one must go beyond the linearized equations of motion to consider finite expectation values. A good example of this behavior are AdS black hole solutions [36]. These are solutions of the full nonlinear supergravity equations of motion, which asymptotically approach AdS space. One can regard the deviations of the metric from the AdS solution as solutions of the linearized gravity equations, and a closer examination shows that these linearized solutions correspond to modes which become singular in the interior of AdS, which simply indicates that the full solutions enter a nonlinear regime.

In the solutions of interest here, there are essentially three excitations in the asymptotically AdS solution: the dilaton, the volume scalar χ and the metric. The massless dilaton couples to the dimension four operator $\mathcal{O}_4$, which is the supersymmetric extension of $\mathrm{Tr}(F^2)$. In eq. (2.2) or (5.6), one finds that asymptotically the

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deviation from its asymptotic value gives $\delta\phi \propto r^{-4}$, in accord with $\delta\psi_-$ in eq. (5.10) for $m^2 = 0$. Hence there is a corresponding expectation value in the dual field theory,

$$\langle\, \mathcal{O}_4 \,\rangle \propto \Delta\, \omega^4\,. \tag{5.11}$$

From eq. (5.4), one can confirm that $m^2L^2 = 32$ for the volume scalar χ. This field couples to the dimension eight operator $\mathcal{O}_8$, which is the supersymmetric extension of $\mathrm{STr}[F_a{}^bF_b{}^cF_c{}^dF_d{}^a - (F_a{}^bF_b{}^a)^2]$ where the STr indicates that the U(N) generators associated with each of the field strengths is symmetrized under the trace [37]. One may extract χ from either the original metric (2.1) or its generalization in eq. (5.9) using eq. (5.2). An asymptotic expansion of the result confirms that $\delta\chi \propto r^{-8}$ in accord with eq. (5.10). The corresponding expectation value is then

$$\langle\, \mathcal{O}_8 \,\rangle \propto (\delta^2 - 4)\, \omega^8\,, \tag{5.12}$$

Finally one may calculate the stress-energy tensor using one of the prescriptions of ref. [35] or [38]. The deviation of the asymptotic metric from AdS space in "nice" coordinates [35] yields $\delta g_{ab} \propto r^{-2}$, and the resulting stress tensor for the general solution (5.9) is

$$\langle\, T_{ab} \,\rangle = \begin{pmatrix} 4(\alpha_1+\alpha_2+\alpha_3)-3\delta & 0 & 0 & 0 \\ 0 & 4\alpha_1-\delta & 0 & 0 \\ 0 & 0 & 4\alpha_2-\delta & 0 \\ 0 & 0 & 0 & 4\alpha_3-\delta \end{pmatrix} \omega^4\,. \tag{5.13}$$

Note that this result is traceless in general, i.e. $\eta^{ab}\langle T_{ab}\rangle = 0$, and for the Lorentz invariant solution (2.1) with $\alpha_1 = \alpha_2 = \alpha_3 = \delta/4$ the stress-energy vanishes completely!

Note that the later seems to be a generic feature of for asymptotically AdS solutions which preserve the SO$(1,p)$ Lorentz symmetry. By the Lorentz symmetry one must have $\langle T_{ab}\rangle = \lambda\eta_{ab}$ for some constant λ. This ansatz yields $\langle T^a_a\rangle = \lambda(p+2)$, however, by the Weyl invariance of the "boundary" field theory, one should also have $\langle T^a_a\rangle = 0$ and hence the stress energy must vanish. We also have explicitly calculated that $\langle T_{ab}\rangle = 0$ for the solutions presented in refs. [16]–[20].

5.4 The role of $\mathcal{O}_8$

In refs. [16, 17] it was suggested that the nontrivial dilaton profile in the solutions constructed there may be triggered by considering the modified theory in which $\mathcal{O}_8$ is added to the $\mathcal{N} = 4$ super Yang-Mills lagrangean. We do not think that this scenario applies for the dual field theory of the present solutions or those considered in ref. [17]. Despite the appearance of a nontrivial dilaton and volume scalar in the present solutions, asymptotically these fields only involve the modes ψ_- and in accord with the standard interpretation, these fields are dual to expectation values of the corresponding operators. Having introduced this scale in the field theory state, it is natural that fermion and scalar masses would be induced through loop effects. Hence the QCD-like behavior uncovered in our analysis would be a natural consequence. Turning on a microscopic coupling i.e., *a constant source current* for $\mathcal{O}_8$, would correspond to exciting in χ the divergent mode $\delta\chi_+ \propto (r/L)^4$, which is absent in the asymptotic expansion of our solutions. The same is, of course, true for the supergravity solutions in ref. [17].

In general, the modes $\delta\psi_-$ decay at the boundary and hence one can expect that the supergravity solutions will remain asymptotically AdS when these modes are excited. That is introducing a finite scale through an expectation value does not greatly disturb the CFT in the UV regime. The same would be true of the modes $\delta\psi_+$ when $m^2 < 0$ which can consistently arise in AdS theories [39]. These supergravity fields are dual to relevant operators in the CFT [3], which again have a negligible effect in the UV regime. Interesting solutions and their interpretation in terms of renormalization group flows have recently appeared in the literature [40]. For $m^2 > 0$, $\delta\psi_+$ is asymptotically divergent and so one can expect that introducing a finite excitation by this mode would destroy the asymptotic AdS structure of the supergravity solution. On the field theory side, this corresponds to introducing an irrelevant operator, which is then not suppressed in the UV regime.

We comment on this because for the volume scalar, we can understand how this occurs, i.e. how $\delta\chi_+$ destroys or modifies the asymptotic structure of the supergravity solution. Let us begin with the full metric (5.5) and only perform the scaling limit (5.8) with large but finite β. That is we do not fully implement the decoupling limit described above. Examining the resulting solution in a regime where $\beta \gg (r^2/L^2) \gg 1$, one finds that the geometry is essentially $AdS_5 \times S^5$ up to small perturbations. That is there is a scaling regime in which the dual theory behaves like a conformal field theory. In particular, examining the volume scalar there is an additional perturbation

$$\delta\chi = \frac{1}{4\beta^2}\frac{r^4}{L^4}\,, \tag{5.14}$$

which corresponds precisely to the mode $\delta\chi_+$ given that $m^2L^2 = 32$ for this scalar. Also calculating curvatures, one finds that the results can be presented in an (double) expansion in $r^2/\beta L^2$ (as well as L^2/r^2) in which the leading terms correspond to those of the AdS geometry.

However, with large but finite β, as we continue to increase r eventually we reach a regime where $r > \beta$ and the perturbative expansion above in $r^2/\beta L^2$ breaks down. Essentially we have again entered a nonlinear regime, now at large r, where the analysis of the linearized equations of motion is insufficient. In the present case, however, we know precisely what the physics of this nonlinear regime is since we have the full supergravity solutions (5.5). In particular, the $\delta\chi_+$ mode does not introduce any divergent curvatures, but it does modify the asymptotic structure of the solution from AdS to that of Minkowski space. So for example in $R_{MN}R^{MN}$, the divergent series constructed with the perturbative expansion above is resummed into the full result which in fact vanishes as $r \to \infty$.

Thus we see that the irrelevant operator $\mathcal{O}_8$ in the CFT controls how the near-horizon geometry expands into asymptotically flat space. This discussion then extends the analysis of ref. [41] where they considered modifying the super Yang-Mills lagrangean by the addition of $\mathcal{O}_8$, and gave evidence that this perturbation allowed one to match scattering cross-sections beyond the near-horizon limit. Of course, this discussion in which we do not fully implement the decoupling limit and how it may be related to introducing a irrelevant operator in the CFT applies in general, and not just to the particular supergravity solutions studied in this paper.

5.5 Consistency and singularities

Let us return to discussing the completely decoupled near-horizon solution, and in particular we will consider the Lorentz invariant solution (2.1) which was the focus of our investigation. Despite the fact that above we argued that this solution should interpreted in terms of introducing expectation values for certain operators in the dual gauge theory, we still ask whether the dual theory deserves to be called a new "phase" of the super Yang-Mills theory. On the one hand, we have a Lorentz invariant state with *zero energy*, as indicated below eq. (5.13). However, we have nontrivial expectation values, e.g., $\langle\mathcal{O}_4\rangle \propto \Delta\omega^4$, which distinguishes this state from the conventional vacuum of the super Yang-Mills theory. There is no immediate contradiction between the nonvanishing expectation value of $\langle\mathcal{O}_4\rangle$ and $\langle T_{ab}\rangle = 0$, even though the expectation values involve linear combinations of the same operators. Because these are normal-ordered composite operators whose expectation values may be positive or negative, delicate cancellations can occur in $\langle T_{ab}\rangle$ and still leave $\langle\mathcal{O}_4\rangle$ nonvanishing — note that it is important that $\eta^{ab}T_{ab} = 0$ for the CFT in order to avoid a contradiction. It seems clear though that this Lorentz invariant zero-energy state can not be a part of the Hilbert space of states built on the conventional *perturbative* vacuum.[2] Hence it would seem justified in referring to the states dual to these supergravity solutions as new phases of the theory. One may wonder if this new phase represents an alternative quantization of the $\mathcal{N} = 4$ super Yang-Mills theory, and if it produces a fully consistent, e.g., stable, theory.

From the supergravity point of view, one may regard these solutions as problematic since they contain naked time-like singularities. Certainly in many cases, string theory is able to resolve singular classical geometries [43]. One should keep in mind, however, that one does not expect string theory to resolve all possible singularities [44]. For example, one does not expect the negative energy Schwarzschild

[2]We advocate this point of view despite the recent discussion of "precursors" [42], since the present system does not involve an localized excitations.

solution (or its AdS cousin, in the present context) to play a role in a fully consistent superstring theory. It would seem that solutions studied here fall into the same category of problematic solutions with time-like singularities and "unusual" energies. It could be that there are various superselection sectors and that the singular solutions are part of a separate sector of the theory, as occurs in the standard Kaluza-Klein framework [45].

Also we note that given smooth nonsingular initial data in the supergravity solution, an asymptotically AdS solution would be protected from evolving into a solution resembling those considered here, by the cosmic censorship conjecture [46]. For example, if one set up a shell of infalling matter which at the same time carried scalar hair, e.g., dilaton or volume scalar excitations, one would expect the hair to be radiated away and that the collapse would produce a conventional black hole. This may be related to the fact that we could not find evidence of a mass gap in the dilaton and metric excitations of the dilaton and metric fields in section 4. That is if the singularity is resolved by adding matter near $r = 0$, it could be that the solution is wildly unstable to radiating away the additional scalar hair. In any event, cosmic censorship would seem to argue in favor of the present solutions representing some exotic new super selection sector of the SYM theory.

Understanding when classically singular background solutions represent physically admissible configurations within string theory is an important and longstanding question. The AdS/CFT correspondence seems to present a new arena in which to to pursue this issue. Of course, the present investigation does not offer an resolution. One may hope though that by examining our backgrounds in combination with other singular asymptotically AdS solutions, some generic features may emerge to produce some new insights into this question. Certainly one feature that the present work reveals is that only examining a limited set of the properties of the singular solutions may mislead one into thinking that the singularity is not relevant for low energy physics. Here we saw that while the mass spectrum of the RR scalar could be reliably calculated, that the dilaton (and the metric) depends on a resolution of the singularity. It would be interesting to examine excitations of a more extensive list of refs. [16]–[20] to see whether or not they are all shielded from the singularity by a potential barrier. We expect generically one will not find such shielding from the singularity [28, 47] for all of the fields (including the massive supergravity and string modes).

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